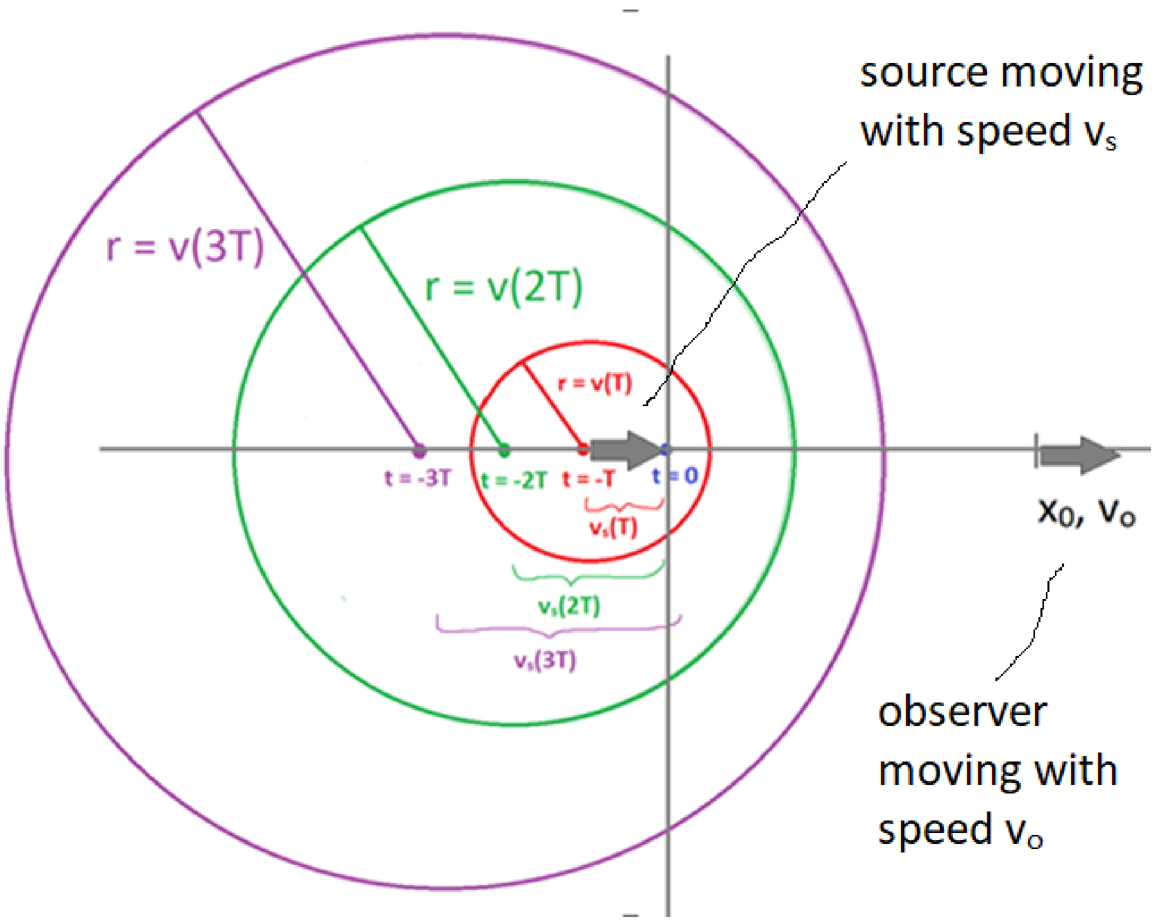
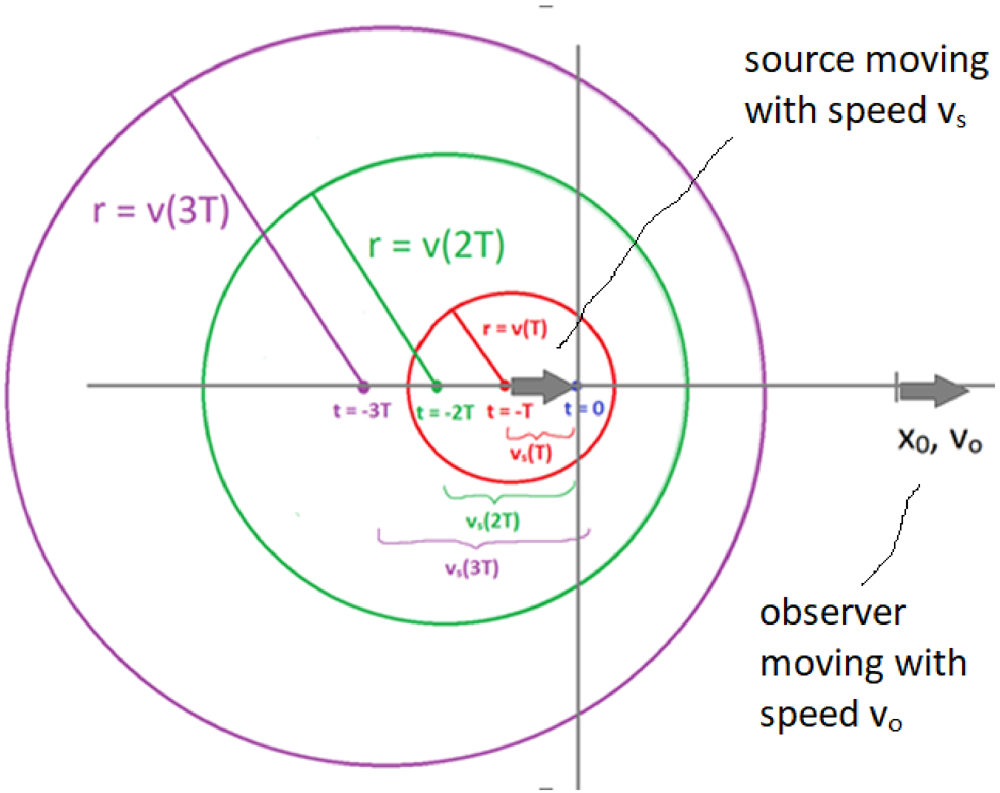
B.3 Doppler Effect



Last thing to discuss is Doppler effect. Consider something like an arrow-shaped ambulance on the left, and an arrow-shaped 'you' on the right. The distance between successive wave crests shrinks, in the direction in which ambulance is moving, and expands in opposite direction. So a *stationary* observer would hear a smaller wavelength (greater frequency) to the right of the ambulance, and would hear a larger wavelength (smaller frequency) to the left of the ambulance.

If observer is moving towards the ambulance, she would intercept the waves more frequently, and thus hear an even higher frequency; if she were moving away, then she'd hear a lower frequency.

B.3 Derivation of Doppler Formula



The observed period would be the amount of time it takes for two successive wave crests, say the blue and red, to intercept the observer. We can get these times from kinematics. Let v be the velocity of the wave.

$$x_o(t) = x_{0(observer)} + v_{(observer)}t \quad = x_0 + v_o t$$

$$x_{blue}(t) = x_{0(blue)} + v_{(blue)}t \quad = 0 + vt = vt$$

$$x_{red}(t) = x_{0(red)} + v_{(red)}t \quad = (-v_s T + vT) + vt$$

Time for red to catch obs.

$$x_{red} = x_o$$

$$(-v_s T + vT) + vt_1 = x_0 + v_o t_1$$

$$t_1 = \frac{x_0 - (v - v_s)T}{v - v_o}$$

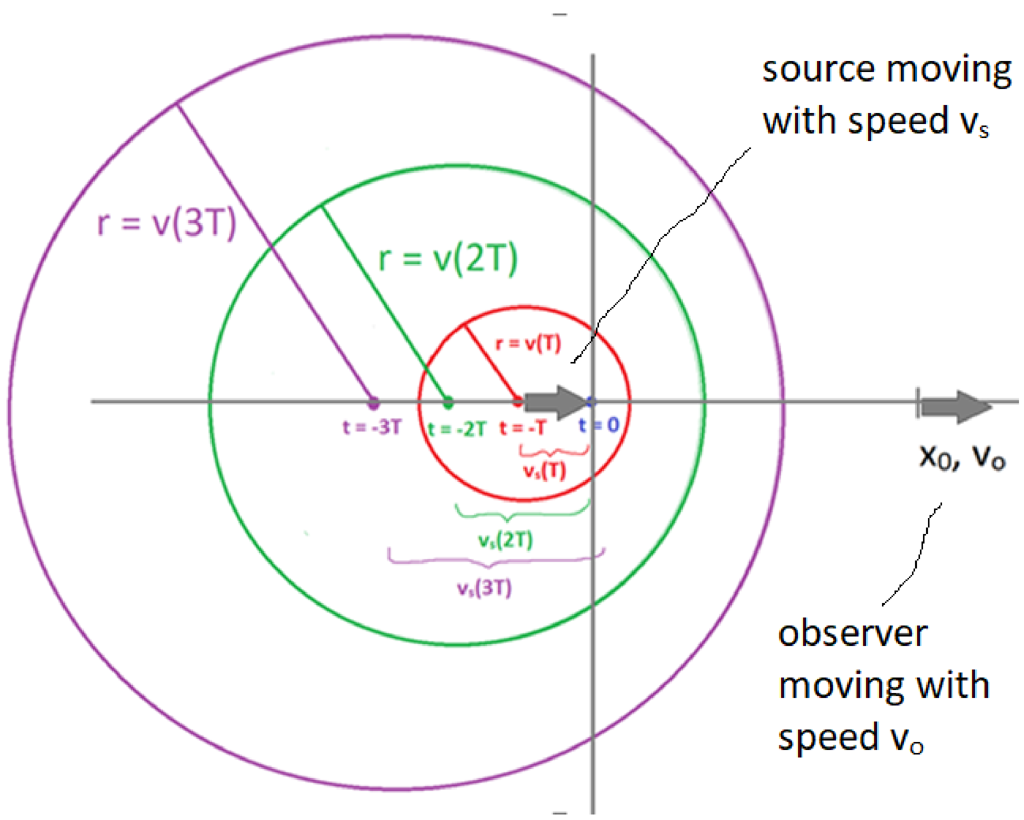
Time for blue to catch obs.

$$x_{blue} = x_o$$

$$vt_2 = x_0 + v_o t_2$$

$$t_2 = \frac{x_0}{v - v_o}$$

B.3 Derivation of Doppler Formula (still)



The difference between these two times is the observed period

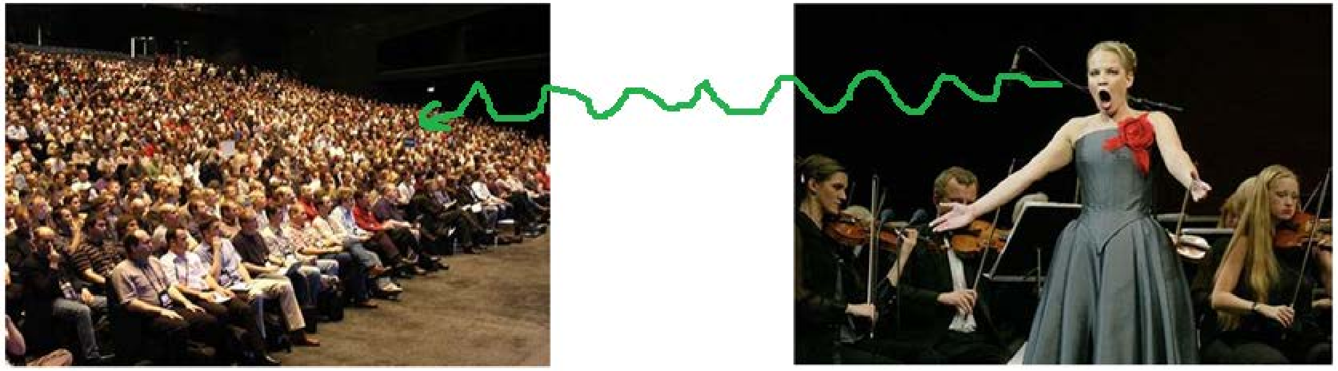
$$
\begin{aligned}
T_o &= t_2 - t_1 \\
&= \frac{x_0 + (v - v_s)T}{v - v_o} - \frac{x_0}{v - v_o} \\
&= \frac{v - v_s}{v - v_o} T
\end{aligned}
$$

Taking the reciprocal of both sides gives us the relationship between the frequencies – i.e. the Doppler equation. And now renaming T as T_s.

$$f_o = \frac{v - v_o}{v - v_s} f_s \qquad \text{Doppler equation}$$

So ‘observe’ (ha!) that if source is approaching, tracking, receding from, observer, f_o will be greater than, equal to, less than f_s

For example…suppose you're an opera singer trying to hit a 1200Hz note, but can only manage 1180Hz. How fast should you run towards your audience so that they hear 1200Hz? You may suppose that your audience isn't running away from *you.* And can suppose speed of sound is 343m/s.

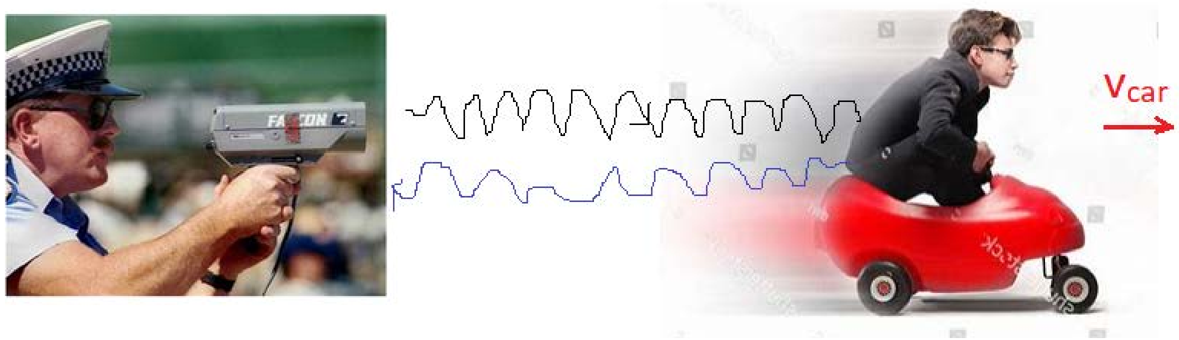


$$f_o = \frac{v - v_o}{v - v_s} f_s$$

$$1200 = \frac{-343 - 0}{-343 - v_s}(1180)$$

$$\frac{1200}{1180} = \frac{-343}{-343 - v_s}$$

$$v_s = -343\left(1 - \frac{1180}{1200}\right) = -5.7\,\mathrm{m/s}$$



And another one. Taken from frequent experience, alas. Suppose you're driving down the road to the right with speed v_{car}. Unbeknownst to you, a cop is covertly tracking your movements, measuring your speed with a Doppler gun which we'll pretend operates on sound waves rather than laser, as is more common now-a-days. Say he emits a pulse 1100Hz and receives it back at 950Hz. How fast were you going?

1. on way out (to the right)

$$f_o = \frac{v - v_o}{v - v_s} f_s$$

$$f_{car} = \frac{343 - v_{car}}{343 - 0}(1100)$$

$$f_{car} = \frac{343 - v_{car}}{343}(1100)$$

2. on way back (to left)

$$f_o = \frac{v - v_o}{v - v_s} f_s$$

$$950 = \frac{-343 - 0}{-343 - v_{car}} f_{car}$$

$$950 = \frac{343}{343 + v_{car}} f_{car}$$

Plug equation 1 into equation 2

$$950 = \frac{343}{343 + v_{car}} f_{car}$$

$$950 = \frac{343}{343 + v_{car}}\left[\frac{343 - v_{car}}{343}(1100)\right]$$

$$\frac{950}{1100} = \frac{343 - v_{car}}{343 + v_{car}}$$

$$(343 + v_{car})\left(\frac{950}{1100}\right) = (343 - v_{car})$$

$$v_{car} = 343\frac{1 - 950/1100}{1 + 950/1100} = 25\,\text{m/s}$$